# ON THE DETAILED FEA OF FLEXIBLE RISERS FOR MULTI-SCALE PROBLEMS

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## ABSTRACT

This paper presents an efficient detailed finite-element modelling method for the structural analysis of flexible risers, which can be effectively implemented in a multiscale analysis based on computational homogenization. For fluid-structure interaction analysis of flexible risers, sufficient accuracy can only be obtained by the use of structural models that properly take into account contact and friction between layers and how these are related to internal and external pressure and bending of individual tendons. With this method, by exploiting cyclic symmetry and applying periodic boundary conditions, only a small fraction of a flexible pipe is used for a detailed nonlinear finite-element analysis at the small scale. In this model, using three-dimensional elements, all layer components are individually modelled and surface-to-surface frictional contact is used to simulate their interaction. The approach is applied on a 5-layered pipe made of inner, outer and intermediate polymer layers and two intermediate armour layers, each made of 40 steel tendons. The capability of the method in capturing the detailed nonlinear effects and the great advantage in terms of significant CPU time saving are demonstrated.

## NOMENCLATURE

| | |
|---|---|
| $R_P$ | Reference point |
| $N_L$ | Left periodic node |
| $N_R$ | Right periodic node |
| L | Riser pipe length |
| $U_i$ | Displacement |
| $\phi_i$ | Rotation |
| E | Young's modulus |
| $\rho$ | Density |
| $\nu$ | Poisson's ratio |
| $e_M$ | Macro Strain |
| $e_m$ | Micro Strain |
| $s_M$ | Macro Stress |
| $s_m$ | Micro Stress |
| $r_0$ | Inner radius |
| $r_1$ | Outer radius |
| PBC | Periodic boundaries |
| RVE | Representative Volume Element |
| R | Rotation matrix |
| $\kappa$ | Curvature |
| $M_B$ | Bending moment |
| Pint | Interal pressure |
| Pext | External pressure |

## INTRODUCTION

Unbonded flexible risers have become the main means for transporting oil and gas between the seabed and surface in ultra-deep waters. They are now critical elements of floating systems for deep water offshore oil and gas production. Flexible risers are capable of functioning in a harsh environment in deeper waters often exceeding 3km below sea level. They are currently used in areas such as Gulf of Mexico, southern shores of Caspian Sea, Norwegian Sea and Greenland. They consist of several polymer and steel layers that are, to a certain extent, free to move internally relative to each other [1]. Their low bending stiffness and their ability of withstanding both horizontal and vertical large displacements make them ideal tools for floating platforms.

The complex structural behavior of flexible risers is not sufficiently understood for many design and development purposes [2]. The complex response of risers to sea wave and current can lead to multi-physics phenomena such as vortex induced vibrations. This can increase the likelihood of fatigue damage and fracture. Sufficient accuracy in many problems in flexible risers, including structure-fluid interaction and vortex induced vibration can only be obtained by the use of non-linear models. These non-linear models should be able to properly take into account contact and friction between layers and determine how these are related to internal and external pressure, bending and torsion of individual tendons, large displacements and rotations. FE models can account for the complex internal structure of flexible risers as shown by de Sousa et al. [3], Merino et al. [4] and Rahmati et al [5]. However, the computational requirements of direct finite element analysis limit their applicability to just a few meters in length at most. The computational cost can be reduced using constitutive laws based on the non-associative elasto-plasticity analogy as shown by Tan et al. [6] and Bahtui et al. [7]. Sævik [8, 9] presented a FE model for predicting stresses due to axisymmetric loads and bending loads in flexible pipes. The interlayer stick-slip behaviour due to friction is taken into account by formulating a constitutive relation based on a plastic beam model, with nonlinear stiffness derived from an analytical formulation in terms of the moment resultants and the wire slips. Experimental studies on the model using strain measurements showed that the numerical distribution of the longitudinal stress predicted by the method is in reasonable agreement with the measured data. A similar approach is used by Alfano et al. [10], building on the analogy between frictional slipping between different layers of a flexible riser and frictional slipping between micro-planes of a continuum medium in non-associative elasto-plasticity. In this way, a linear elastic relationship was used for the initial response, in which no slip occurs, and a non-associative rule with linear kinematic hardening was then introduced to model the full-slip phase. The determination of the parameters of the constitutive law to bridge the small scale of the detailed FE simulations with the large scale of the model is a challenge. An alternative approach which does not have this limitation is a fully-nested multi-scale procedure [11], currently in widespread use for the modelling of composite materials. With this method, at each integration point (i.e. cross section) of the large-scale beam model, the stress resultants corresponding to assigned generalised strains are determined through the solution of the small-scale FE problem.

This paper describes an efficient modelling approach for the small-scale analysis, which exploits the cyclic symmetry of the riser detailed structure, and its implementation based on the introduction of periodic boundary conditions for detailed FE models in small scale simulations. In the following sections the computational homogenization method which is based on the work of Edmans et al. [12, 13] is explained and the numerical results and CPU time obtained by using this model are reported and discussed to evaluate the accuracy and the computational saving entailed by the use of periodic boundary condition in this method.

## MULTISCALE ANALYSIS OF RISERS

A fully-nested computational homogenization scheme is essentially based on the construction of a micro-scale (or more generally small-scale) boundary-value problem (BVP) at each integration point of a macro (or large-scale) model. The small-scale model is solved numerically to determine the constitutive response of the material at each integration point. If the FEM is used at both scales, the methods is also known as $FE^2$. If a displacement-based FE formulation and a Newton-Raphson incremental scheme are used at the large scale, 'tentative' strains resulting from an attempted displacement increment are calculated at each integration point for each iteration of each increment in the macro model.

When a 3D continuum model is used at both scales, the tentative strains are imposed on a suitably defined representative volume element (RVE) of the micro scale (down-scaling procedure), each large-scale integration point corresponding to one and only RVE. To this end, it is assumed (a) that the displacement field at the small scale, $u_m$, is composed of a smooth (linear within the RVE) part $v_m$, equal to the large-scale field $v_M$, and a fluctuation field $w_m$:

$$u_m = v_m + w_m \qquad v_m = v_M \tag{1}$$

(b) that the macro strain at each integration point, $\varepsilon_M$, is given by the symmetric part of the gradient of $v_m$:

$$\varepsilon_M = \nabla^s v_M = \nabla^s v_m \tag{2}$$

(c) and that the average of the strain tensor in the RVE is equal to the strain tensor at the integration point:

$$\varepsilon_M = \frac{1}{\Omega_{RVE}} \int_{\mathrm{V}_{RVE}} \varepsilon_m \, d\mathrm{V}_{RVE} \tag{3}$$

where $\mathrm{V}_{RVE}$ denotes the volume of the RVE. Using the Green theorem, this results in a general relation at the boundary that can be satisfied by a number of different boundary conditions, from the 'stiffest' ones of zero fluctuations within the RVE to the 'most compliant' of uniform tractions at the boundaries of the RVE; in the majority of cases the most effective and widely used are the periodic boundary conditions. The resulting macro stresses at the considered integration point are obtained by averaging the stresses over the RVE at the micro scale (up-scaling procedure), which is equivalent to the enforce that the virtual work for arbitrary variations of macro strains is the same at the large and the small scales (Hill condition), see [9] and references therein.

For the flexible risers considered in this paper, a 3D continuum model is used at the small scale, which is to be linked to a large-scale beam model, where generalised strains and stress resultants are employed. This means that different structural models are used at different scales whereby $v_m = v_M$ in equation (1) and the assumption the strain averaging assumption of equation (3) cannot be made as they make no sense. For this reason an extension of the theory has been proposed by Edmans et al. [9]. In this extended theory, a Representative Domain Element (RDE) instead of RVE is used, to emphasize that the models involved do not necessarily need to be solid continuum ones. On the other hand, in this paper the model at the small scale is indeed a continuum one, so the acronym RVE will be still used. Referring to the original paper for the details of the derivation, a geometrically non-linear formulation is assumed at the large scale, but it is assumed that displacement and rotations are large but macro strains are small enough so that a geometrically linear formulation can be assumed at the small scale. The small-scale problem is then written in its general form as follows:

$$\begin{cases} Q_{bc}u_m = Q_{bc}\overline{P}\,\varepsilon_M \\ \langle\, \sigma_m(B_m u_m), B_m \delta u_m \,\rangle = 0 \end{cases} \tag{4}$$

$$\forall \delta u_m : Q_{bc}\delta u_m = 0$$

In this equation, $\overline{P}$ is a linear operator that translates the large-scale strain $\varepsilon_M$ at the integration point into a corresponding small-scale displacement field $v_m$, $Q_{bc}$ is a suitably defined operator that extracts the appropriate boundary values of a small-scale displacement field, so that Equation $(4)_1$ represents the boundary conditions on the RVE; $B_m$ is the linear operator mapping the micro strains to the micro displacement field; $\sigma_m$ is the micro stress field and $\sigma_m(\varepsilon_m)$ denotes the constitutive equation at the small scale, that in the case of a flexible riser is in general nonlinear because of the unilateral frictional contact between the different components of the flexible riser; finally, symbol $\langle\, x, y\rangle$ in Equation $(4)_2$ represents the virtual work computed by a stress field $x$ for a virtual strain field $y$. Therefore, Equation $(4)_2$ is the variational enforcement of equilibrium in the RVE, which is done approximately here because a FE model is used at the small scale.

Given a large-scale strain $\varepsilon_M$, the small-scale problem consists of finding a small-scale displacement field $u_m$ solution of the variational problem (4). This defines a nonlinear operator P mapping a displacement field $u_m \in V_m$ to any given large-scale strain $\varepsilon_M \in D_M$, $V_m$ and $D_M$ being two suitably defined linear spaces of small-scale displacement and large-scale strain fields. Once $u_m$ has been determined, the small-scale strain is given by $\varepsilon_m = B_m u_m$, and the small-scale stress is given by the application of the constitutive law, $\sigma_m(\varepsilon_m)$.

The large-scale stress at the considered integration point, $\sigma_M$, is then determined (up-scaling procedure) in a variational way, through a 'generalised Hill condition' (GHC) enforcing the equality of the virtual works at the two scales (see Figure 1), as follows:

$$\langle \sigma_M, \delta\varepsilon_M \rangle = \langle \sigma_m, B_m \delta u_m \,\rangle \qquad \forall\ \delta\varepsilon_M \epsilon\ D_M$$
$$\forall\ \delta u_m : Q_{bc}\delta u_m = Q_{bc}\overline{P}\ \delta\varepsilon_M \tag{5}$$

In practice, for each RVE a dummy reference node $R$ is introduced, whose degrees of freedom are collected in a vector $\eta_{MR}$, which therefore represent the components of the macro strain. Therefore Equation (4) and (5) become:

$$\begin{cases} Q_{bc}u_m = Q_{bc}\overline{P}\,\eta_{MR} \\ \langle\, \sigma_m(B_m u_m), B_m \delta u_m \,\rangle = 0 \end{cases} \tag{6}$$

$$\forall \delta u_m : Q_{bc}\delta u_m = 0$$

$$\langle \sigma_M, \delta\eta_{MR} \rangle = \langle \sigma_m, B_m \delta u_m \,\rangle \qquad \forall\ \delta\eta_{MR} \epsilon\ D_M \tag{7}$$
$$\forall\ \delta u_m : Q_{bc}\delta u_m = Q_{bc}\overline{P}\ \delta\eta_{MR}$$

which shows that $\sigma_M$ is simply obtained in practice as the nodal reaction vector at the dummy reference node.

In this way, for example, prescribing a rotation $\phi_{R_P}$ of the reference point about one axis in the cross section is equivalent to prescribing periodic boundary conditions corresponding to a bending curvature equal to $\phi_{R_P}/L$, $L$ being the length of the model.

In other words, the vector $\eta_{MR}$ appearing in Equation (6) and, in its variation, in Equation (7) is given by:

$$\eta_{MR} = \begin{bmatrix} \boldsymbol{U}_{R_P} \\ \boldsymbol{\phi}_{R_P} \end{bmatrix} = \begin{bmatrix} U_{R_P x} \\ U_{R_P y} \\ U_{R_P z} \\ \phi_{R_P x} \\ \phi_{R_P x} \\ \phi_{R_P x} \end{bmatrix} \tag{8}$$

Following [12], for each one of these pairs of nodes, a new 'dummy' projected node is introduced on a plane which is parallel to the end cross section in their un-deformed configuration. This is shown in Figure 2, where, for the generic pair of nodes, $n_L$ and $n_R$ indicate the corresponding nodes on the left-hand and right-hand end cross section, respectively, and $n_P$ denotes the corresponding projected node.

A set of linear constraint equations was then generated, relating the degrees of freedom of each pair of nodes on the two end cross section to those of the corresponding projected node. The link is enforced for all displacement components $U_n^i$ and rotation components $\phi_n^i$ of a node $n$, as follows:

$$U_{n_L}^i - U_{n_R}^i = U_{n_P}^i \qquad i=1,2,3 \tag{9}$$

$$\phi_{n_L}^i - \phi_{n_R}^i = \phi_{n_P}^i \qquad i=1,2,3 \tag{10}$$

The displacement and rotation vectors, $\boldsymbol{U}_{n_P}$ and $\boldsymbol{\phi}_{n_P}$ of each dummy projected node $n_P$ are rigidly constrained to the displacement and rotation vectors, $\boldsymbol{U}_{R_P}$ and $\boldsymbol{\phi}_{R_P}$, of a projected reference point $R_P$ at their center, using the following rigid-body constraint equations:

$$\boldsymbol{U}_{n_P} = \boldsymbol{U}_{R_P} + \boldsymbol{\phi}_{R_P} \times \boldsymbol{X}_{n_P} \tag{11}$$

$$\boldsymbol{\phi}_{n_P} = \boldsymbol{\phi}_{R_P} \tag{12}$$

where $\boldsymbol{X}_{n_P}$ indicates the position vector of the projected node $n_P$ with respect to the reference point and $\times$ denotes the standard cross product of two vectors.

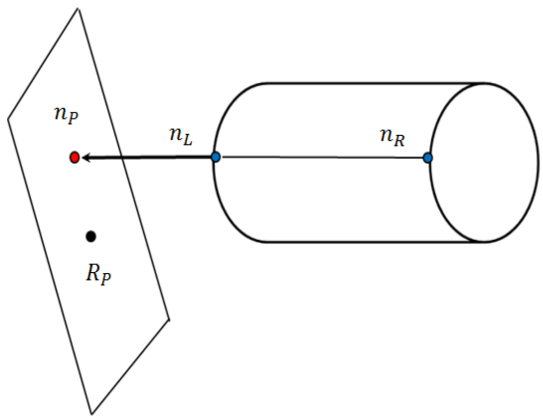


Figure 1: Correspondence between a pair of nodes $n_L$ and $n_R$ on the two end-cross sections and the dummy projected node $n_P$, which is constrained to the reference node $R_P$.

## DETAILED FINITE-ELEMENT MODELS OF SEGMENTS OF RISER

A simplified 5-layer flexible pipe, made of three polymer layers and two armour layers, was considered. Both the inner and the outer armour layers are made of 40 steel tendons, with rectangular cross section, which are wound with the same pitch length $L_p$ equal to 320mm. The model is created using the FE package ABAQUS [14], version 6.13.1.

Table 1: Length and number of nodes and elements of the models.

| Model | L (mm) | No. of Nodes | No. of Elements |
|---|---|---|---|
| Lp/40 | 8 | 7901 | 2580 |
| Lp | 320 | 225852 | 100320 |

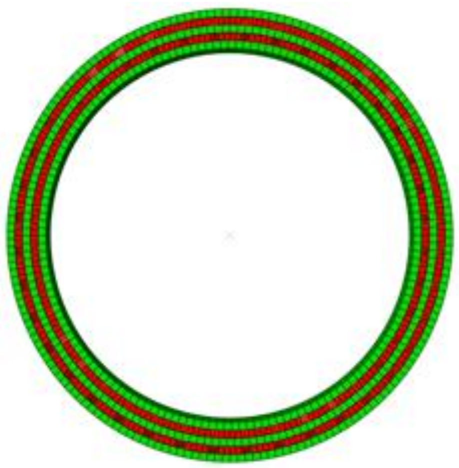

Figure 2 – Cross section of the FE model.

All components are modelled with fully-integrated 8-noded 3D solid elements with incompatible strains, with surface-to-surface frictional contact between all components. A cross section of the model is shown in Figure 1. The model has a length equal to 1/40 of the pitch length of the tendons (Lp/40). Details of the material, dimensions and arrangement of constituent layers are given in Tables 1 and 2. Periodic boundary conditions are applied for the small scale model, see [5] for more details on implementing boundary condition.

Table 2: Dimensions and materials of components in the model.

| Layer | $r_0$, $r_1$ (mm) | Material | E(MPa) | v |
|---|---|---|---|---|
| 1 | 48, 50 | Polyethylene | 0.35 | 0.4 |
| 2 | 50, 52 | Carbon Steel | 210 | 0.3 |
| 3 | 52, 54 | Polyethylene | 0.35 | 0.4 |

## NUMERICAL RESULTS

The analyses were conducted by applying internal and external pressure in a first step, after which one symmetric cyclic history of bending curvature was prescribed, the maximum and minimum curvatures being 0.125 and -0.125 $m^{-1}$. The curvature can be obtained from the large scale model while the pressure and external force for the large scale model can be either provided by separate analyses and data, or obtained using the flow solver within a multi-physics fluid-structure interaction analysis.

Three cases are shown in figure 3 where the internal pressure varies from 2 to 8 MPa. It is shown in figure 3 that there is a strong nonlinear relation between the bending meoment and the curveture. Moreover this nonlinear relation can be significantly influenced by the values of internal and external pressures. In a fully coupled fluid-structure interaction model these internal and/or external pressures are usually obtained from a fluid solver after each iteration. Therfore, sufficient accuracy in a structure-fluid interaction problem for flexible risers cannot be obtained if the effects of pressure on the bending stiffness is not taken into account. Figure 4 highlights the huge saving in CPU time, from 11560 minutes for the longest model of one pitch length (Lp) to only few minutes for the smallest model (Lp/40) used in this study.

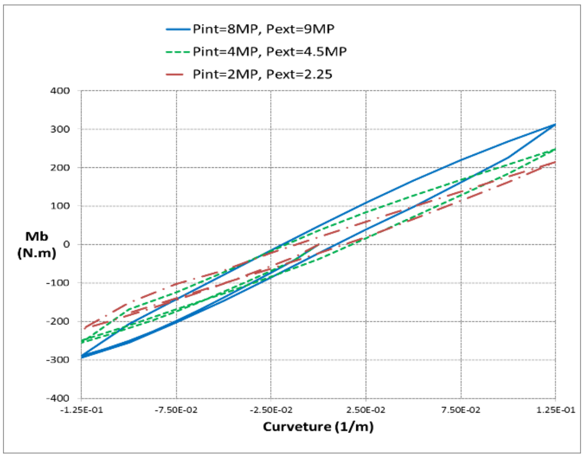


Figure 3 – Bending-moment vs curvature for three values of the external pressure.

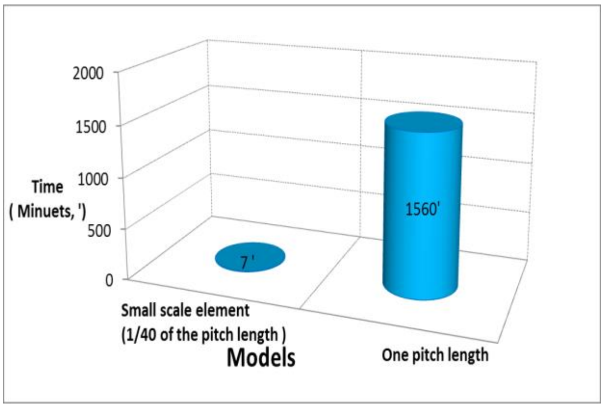


Figure 4 – Computational cost of the model compared with a riser with one pitch length.

## CONCLUSIONS

An approach to minimize the computational cost of a detailed nonlinear FE analysis of a segment of flexible riser, which includes all layers and accounts for frictional contact between them is presented. This method can be effectively used as the small-scale model in a nested multi-scale analysis. For multi-physics probelms such as fluid-structure interaction analysis of flexible risers, sufficient accuracy can be obtained by the use of this method in which the contact and friction between layers and their relations to the internal and external pressure and bending of individual tendons are taken into account.The key ideas behind the proposed approach are (a) the observation that flexible risers can be represented, with a very good approximation, with a model having cyclic symmetry, (b) the use of such cyclic symmetry to reduce the length of the model to the smallest repeating unit and (c) the use of periodic boundary conditions. The enormous saving in computational cost entailed by the use of the smallest repeating unit makes this the optimal model to be used in a nested ($FE^2$) multiscale analysis and in a fluid-structure solution method.


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